Find solutions to

$$\begin{cases} u+v+w &= 1 \\ v+w &= 1 \\ w &= 1 \end{cases}$$

We can view this as a matrix equation

$$\begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

- The coefficient matrix is upper-triangular.
- Solve by back substitution.

Inverses

Let's go back to our example

$$\begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

Multiply by the inverse

$$\begin{pmatrix} 1 & -1 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}$$

we'll get $(u, v, w)^T = (0, 0, 0)^T$.

Q: What if we change $(0, 0, 0)^T$ to $(-1, 0, 1)^T$.

Q: How many solutions do we get?

Find two solutions to

$$\begin{cases} u+w &= 0 \\ u+v &= 0 \\ v-w &= 0 \end{cases}$$

The matrix equation to this system is

$$\begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & -1 \end{pmatrix} \begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

Notice that this leads to

- Eqn. 1: $u = -w$
- Eqn. 2: $u = -v$
- Eqn. 3: $u = w$

Dependent rows lead to redundant equations.

Two solutions to the system are $(1, -1, -1)^T$ and $(0, 0, 0)^T$.

- Are there only two solutions?
- How many solutions are there?
- Can we characterize the solution set?

$$\begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix} t = \begin{pmatrix} t \\ -t \\ -t \end{pmatrix}$$

Independent & Dependent

- If two vectors are independent - plane
- If two vectors are dependent - line
- For three vectors?

- What does it mean for two vectors to be independent?
- What does it mean for two vectors to be dependent?
- Go back to the system which had many solutions and note what the dependency is.

Elimination Matrices

$$E = \begin{pmatrix} 1 & 0 \\ -l & 1 \end{pmatrix} \qquad E^{-1} = \begin{pmatrix} 1 & 0 \\ l & 1 \end{pmatrix}$$

We can even derive this through solving a system of equations.